Rules and Properties of Integers for the GMAT

Even and Odd Integers

Units digit for even numbers: 0, 2, 4, 6, 8
Units digit for odd numbers: 1, 3, 5, 7, 9

- Odd + or - odd = even
- Even + or - even = even
- Odd + or - even = odd
- Odd × odd = odd
- Even × even = even
- Odd × even = even

Negative and Positive Integers

- Adding a negative integer is the same as subtracting the positive
- Subtracting a negative integer is the same as adding the positive (double negatives cancel out)
- Negative × or / negative = positive
- Negative × or / positive = negative
- Positive × or / negative = negative
- Positive × or / positive = positive

Multiples and Factors

Multiple/Factor = Integer

If \( x \) and \( y \) are integers and \( y = xn \) for some integer \( n \), then \( x \) is a factor (or divisor) of \( y \), and \( y \) is a multiple of \( x \).

- greatest common factor, or greatest common divisor (GCF and GCD), is the largest factor shared by two numbers.
- The least common multiple (LCM) is the lowest multiple that two factors share.

Quotients and Remainders

- The remainder is what is left over in a division problem.
- The quotient is how many times an integer can fit into another integer, regardless of what’s left over.
- \( y = xq + r \), and \( 0 \leq r < x \)
Properties of the Integer 0

- Even
- Neither positive nor negative
- Any number plus or minus 0 is that number: \( n + 0 = n; n - 0 = n \)
- Any number multiplied by 0 is 0: \( n \times 0 = 0 \)
- You cannot divide by 0: \( n/0 = \text{undefined} \)

Properties of the Integer 1

- Odd
- Any number multiplied or divided by 1 is itself: \( n \times 1 = n; n / 1 = n \)
- Any number other than zero divided by itself is 1: \( n/n = 1, \text{ if } n \neq 0 \)
- The reciprocal of any number is 1/that number. Any number other than zero multiplied by its reciprocal equals 1: \( n \times 1/n = 1, \text{ if } n \neq 0 \)
- Multiplying a number by -1 changes the sign but not the absolute value.

Prime Numbers

- Prime numbers are positive integers that can only be divided by itself and 1
- The first prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

Prime Factorization

- Breaking down a number into its prime factors
- Any multiple, by definition, will include in its own prime factorization the prime factorization of its factors. The factors’ prime factorizations will show up in the multiple’s prime factorization.

Consecutive Integers
Consecutive integers: \( n + 1, n + 2, n + 3 \ldots, \) where \( n \) is an integer

- If the number of integers in a consecutive set is odd: the sum of all the integers is \textit{always} divisible by that number.
- If the number of integers in a consecutive set is even, the sum of the integers is \textit{never} divisible by that number.
- If \( n \) is an integer greater than 1, then \( n \) factorial, represented by the symbol \( n! \), is the \textit{product of all of the integers from 1 to} \( n \).
- The product of \( n \) consecutive integers is \textit{always} divisible by \( n! \)

Consecutive even integers: \( 2n, 2n + 2, 2n + 4, 2n + 6 \ldots, \) where \( n \) is an integer

Consecutive odd integers: \( 2n + 1, 2n + 3, 2n + 5, 2n + 7 \ldots, \) where \( n \) is an integer
Factorials

- If \( n \) is an integer greater than 1, then \( n \) factorial, represented by the symbol \( n! \), is the product of all of the integers from 1 to \( n \).
- \( 0! \) is the same as \( 1! = 1 \)

Exponents and Square Roots

If the square root of a given number is an integer, that means that the number has to be an integer too and is a perfect square. Perfect squares are \( 1 \) (\( 1^2 \)), \( 4 \) (\( 2^2 \)), \( 9 \) (\( 3^2 \)), \( 16 \) (\( 4^2 \)), and so on.