PrepScholar

# Rules and Properties of Integers for the GMAT

## Even and Odd Integers

Units digit for even numbers: 0, 2, 4, 6, 8
Units digit for odd numbers: 1, 3, 5, 7, 9

- Odd + or - odd = even
- Even + or - even = even
- Odd + or - even = odd
- Odd × odd = odd
- Even × even = even
- Odd × even = even

## Negative and Positive Integers

- Adding a negative integer is the same as subtracting the positive
- Subtracting a negative integer is the same as adding the positive (double negatives cancel out)
- Negative × or / negative = positive
- Negative × or / positive = negative
- Positive × or / negative = negative
- Positive × or / positive = positive

## Multiples and Factors

Multiple/Factor = Integer

If $x$ and $y$ are integers and $\boldsymbol{y = xn}$ **for some integer** $\boldsymbol{n}$, then $x$ is a **factor** (or divisor) of $y$, and $y$ is a **multiple** of $x$.

- **greatest common factor**, or greatest common divisor (GCF and GCD), is the largest factor shared by two numbers.
- The **least common multiple** (LCM) is the lowest multiple that two factors share.

## Quotients and Remainders

- The **remainder** is what is left over in a division problem.
- The **quotient** is how many times an integer can fit into another integer, regardless of what's left over.
- $y = xq + r$, and $0 \leq r < x$

## Properties of the Integer 0

- Even
- Neither positive nor negative
- Any number plus or minus 0 is that number: **n + 0 = n**; **n - 0 = n**
- Any number multiplied by 0 is 0: **n × 0 = 0**
- You cannot divide by 0: **n/0 = undefined**

## Properties of the Integer 1

- Odd
- Any number multiplied or divided by 1 is itself: **n × 1 = n, n / 1 = n**
- Any number *other than zero* divided by itself is 1: **n/n = 1, if n ≠ 0**
- The **reciprocal** of any number is 1/that number. Any number *other than zero* multiplied by its reciprocal equals 1: **n × 1/n = 1, if n ≠ 0**
- Multiplying a number by -1 changes the sign but not the **absolute value**.

## Prime Numbers

- Prime numbers are positive integers that can only be divided by itself and 1
- The first prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

## Prime Factorization

- Breaking down a number into its prime factors
- Any multiple, by definition, will include in its own prime factorization the prime factorization of its factors. The factors' prime factorizations will show up in the multiple's prime factorization.

## Consecutive Integers

Consecutive integers: n + 1, n + 2, n + 3 …, where n is an integer

- **If the number of integers in a consecutive set is odd: the sum of all the integers is *always* divisible by that number.**
- **If the number of integers in a consecutive set is even, the sum of the integers is *never* divisible by that number.**
- If n is an integer greater than 1, then n **factorial**, represented by the symbol n!, is **the product of all of the integers from 1 to n**.
- **The product of n consecutive integers is always divisible by n!**

Consecutive even integers: 2n, 2n + 2, 2n + 4, 2n + 6 …, where n is an integer

Consecutive odd integers: 2n + 1, 2n + 3, 2n + 5, 2n + 7 …, where n is an integer

## Factorials

- If n is an integer greater than 1, then n **factorial**, represented by the symbol n!, is **the product of all of the integers from 1 to n**.
- 0! Is the same as 1! = 1

## Exponents and Square Roots

If the square root of a given number is an integer, that means that that the number has to be an integer too and is a **perfect square**. Perfect squares are 1 ($1^2$), 4 ($2^2$), 9 ($3^2$), 16 ($4^2$), and so on.